Peristaltic motion of a Micropolar fluid under the effect of a magnetic field in an inclined channel

1S. V. H. N. Krishna Kumari. P, 2Saroj D Vernekar, 3Y. V. K. Ravi Kumar
1Department of Mathematics, Auroras Scientific Technological and Research Academy, HYDERABAD, India.
2Department of Mathematics, Methodist College of Engineering and Technology, HYDERABAD, India.
3Practice School Division, Birla Institute of Technology and Sciences(BITS) – Pilani, HYDERABAD, India.

ABSTRACT
Mathematical modeling and numerical solution are presented for the flow of a Micropolar fluid under the effect of magnetic field in an inclined channel of half width 'a' under the considerations of low Reynolds number. A long wavelength approximation is used to solve the flow problem. The effect of various parameters on the pumping characteristics is discussed through graphs.

Keywords – Peristalsis, Micropolar fluid, Magnetic field, inclined channel.

I. INTRODUCTION
Peristaltic pumping is a form of fluid transport, generally from a region of lower to higher pressure, by means of a progressive wave of area contraction or expansion which propagates along the length of a tube like structure. Peristalsis occurs naturally as a means of pumping biofluids from one place of the body to another. This mechanism occurs in the gastrointestinal, urinary and reproductive tracts and many other glandular ducts in the living body. The early reviews of Ramachandra Rao and Usha [1] , Jaffrin and Shapiro[2] , Manton [3], Brasseur et al. [4], Srivastava and Srivastava [5], Provost and Schwarz [6], Shukla and Gupta [7], Misra and Cakmak [22] discussed three basic viscous flows of micropolar fluids. They are Couette and Poiseuille flows between two parallel plates and the problem of a rotating fluid with a free surface. The results obtained are compared with the results of the classical fluid mechanics. Srinivasacharya et al.[23] made a study on the peristaltic pumping of a micro polar fluid in a tube.

Magnetohydrodynamics (MHD) is the science which deals with motion of highly conducting fluids in the presence of a magnetic field. The motion conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise the mechanical forces which modify the flow of the fluid.(Ferraro V C A,[24]). The effect of moving magnetic field on blood flow was studied by Stud et al (Stud et al.[25]), and they observed that the effect of suitable moving magnetic field accelerates the speed of blood.

Krishna Kumari et.al [26] studied the peristaltic pumping of a Casson fluid in an inclined channel under the effect of a magnetic field. Krishna Kumari et.al [27] studied the peristaltic pumping of a Jeffreyy fluid in a porous tube. Ravi Kumar et.al [28] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. However, the peristaltic transport of micropolar fluids in an inclined channel in the presence of magnetic field has not been studied.
In view of this, we considered the peristaltic pumping of a micropolar fluid in an inclined channel under the effect of magnetic field. This mathematical model may be useful to have a better understanding of the physiological systems such as blood vessels.

II. MATHEMATICAL FORMULATION

Consider the peristaltic pumping of a micropolar fluid in an inclined channel of half-width ‘a’. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity we restrict our discussion to the half-width of the channel as shown in Fig. 1. The wall deformation is given by

\[ H (X, t) = a + b \sin \left( \frac{2\pi}{\lambda} (X - ct) \right) \]  

where \( b \) is the amplitude, \( \lambda \) is the wavelength and \( c \) is the wave speed.

Under the assumption that the channel length is an integral multiple of the wavelength \( \lambda \) and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame \((x, y)\) moving with velocity \( c \) away from the fixed (laboratory) frame \((X, Y)\). The transformation between these two frames is given by

\[ \begin{align*} 
  x &= X - ct; 
  y &= Y; 
  u(x, y) = U(X - ct, Y); 
  v(x, y) = V(X - ct, Y) 
\end{align*} \]  

where \( U \) and \( V \) are velocity components in the laboratory frame and \( u, v \) are velocity components in the wave frame. In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. So, we assume that the flow is inertia-free. Further, we assume that the wavelength is infinite.

Using the non-dimensional quantities,

\[ \begin{align*} 
  u &= \frac{u}{c}; 
  x = \frac{x}{\lambda}; 
  y = \frac{y}{a}; 
  p &= \frac{pa}{\mu}; 
  \Omega = \frac{\Omega a}{c}; 
  \eta = \frac{H}{a} 
\end{align*} \]

The non-dimensional form of equations governing the motion (dropping the bars) are

\[ \begin{align*} 
  \frac{\partial^2 u}{\partial y^2} + N \frac{\partial \Omega}{\partial y} (u - 1)M^2 - (1 - N) \frac{\partial p}{\partial x} + \eta \sin \theta &= 0 
  \end{align*} \]  

\[ \begin{align*} 
  2 - N \frac{\partial^2 \Omega}{\partial y^2} - \frac{\partial u}{\partial y} - 2\Omega &= 0 
  \end{align*} \]

where \( N = \frac{k}{\mu + k} \) is coupling number, \( \Omega \) is the micro rotation velocity, \( u \) is the velocity, \( \mu \) is the viscosity of the fluid, \( k \) is the micropolar viscosity, \( M \) is the micropolar parameter, \( p \) is the fluid pressure, and \( M \) is the Hartmann number.
The corresponding non-dimensional boundary conditions are

\[ \frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \quad (5) \]
\[ \frac{\partial \Omega}{\partial y} = 0 \quad \text{at } y = 0 \quad (6) \]
\[ u = -\frac{\sqrt{Da}}{\alpha} \frac{\partial u}{\partial y} \quad \text{at } y = h(x) \quad (7) \]
\[ \Omega = -1 - \frac{\sqrt{Da}}{\alpha} \frac{\partial u}{\partial y} \quad \text{at } y = h(x) \quad (8) \]

where Da is the Darcy number and \( \alpha \) is the slip parameter.

**III. SOLUTION OF THE PROBLEM**

The general solution of (3) and (4) using the boundary conditions (5)-(8) is given by

\[ u = L_{10} \cosh(L_{1} y)(L_{4} L_{14} + L_{7} c_{1} + \frac{L_{3}}{L_{1}} e^{L_{1} y} (L_{14} - 1) + (e^{L_{1} y} + e^{-L_{1} y}) (L_{14} + 1) c_{1} + L_{10} (L_{14} L_{4} \cosh(L_{1} y)) + \frac{L_{3}}{L_{5}} (e^{L_{1} y} + e^{-L_{1} y}) \frac{h_{1}}{M}) \]

\[ \Omega = L_{13} L_{14} \sin(L_{1} y)(L_{4} - L_{7} (L_{4} L_{14} + L_{7} c_{1}) (L_{14} + 1) C_{1} + L_{13}) + L_{1} \left(L_{4} - L_{7} \right)(L_{14} L_{4} + 1) C_{1} + L_{13} \right) e^{L_{1} y} \]

\[ + L_{4} \left(L_{4} - L_{7} \right)(L_{4} e^{L_{1} y} + L_{4} (L_{4} - L_{7} ) e^{-L_{1} y} \right) \]

\[ N_{1} = M_{z}^{2} + (1 - N) \frac{\partial p}{\partial x} - \eta \sin \theta; \quad \Omega_{x} = \sqrt{\frac{M_{z}^{2} + m_{z}^{2} + \frac{1}{2} \left(2(M_{z}^{2} - m_{z}^{2})^{2} - N \left(M_{z}^{2} + m_{z}^{2}\right)^{2}\right)}{2 - N}}; \quad \Omega_{x} = \frac{M_{z}^{2} - NL_{z}}{2}; \quad L_{n} = 1 / 2 NL_{z}; \]

\[ L_{14} = L_{14} \left(L_{14} L_{14} - L_{7}^{2}\right); \quad L_{7} = L_{4} (L_{4} L_{14} + L_{7} c_{1}) (L_{14} + 1); \quad L_{14} = L_{4} (L_{4} + L_{7}) + (L_{7} - L_{7}^{2}); \quad L_{10} = L_{4} L_{10} \left(L_{10} \cosh(L_{10} h) - \sinh(L_{10} h) \right); \]

\[ c_{1} = (-1 / L_{25} L_{26} - L_{25} - L_{19} L_{23} + L_{24}) ; \quad L_{25} = L_{20} + L_{21} - L_{22} \]
The volume flux \( q \) through each cross section in the wave frame is given by

\[
q = \int_0^h udy
\]

The pressure gradient is obtained from equation (11)

\[
\frac{\partial p}{\partial x} = (M^2 - (1 - N))[ (q - S_m)/(S_2 - hL_{12})] L_{12} - (M^2 + \eta \sin \theta)(1 - N);
\]

\[
(12)
\]

\[
(\partial^2 p/\partial x^2) \sinh (L_3 h), \quad S_2 = L_4 ((e^{L^h} - 1)/L_1), \quad S_1 = (e^{L^h} - 1)/L_1, \quad S_4 = S_1 L_{24} - S_2 + S_5, \quad S_6 = S_4 (1 - L_{44})/L_5 L_{25}.
\]

The time averaged flow rate is given by

\[
\bar{Q} = q + 1
\]

IV. THE PUMPING CHARACTERISTICS

Integrating the equation (12) with respect to over one wave length, we get the pressure rise (drop) over one cycle of the wave as

\[
\Delta p = \int_0^1 \frac{\partial p}{\partial x} dx
\]

(14)

The dimensionless frictional force \( F \) at the wall across one wavelength in the inclined channel is given by

\[
F = \int_0^1 h \left( - \frac{\partial p}{\partial x} \right) dx
\]

(15)

V. RESULTS AND DISCUSSIONS

The variation of pressure rise \( \Delta p \) with time averaged flow rate for different values of \( \alpha \) is shown in Fig.2. It is observed that for a given \( \bar{Q}, \Delta p \) decreases as the slip parameter \( \alpha \) increases in the pumping and free pumping regions. The opposite behavior is observed in co-pumping region. And also for a given \( \Delta p \) the flux \( \bar{Q} \) increases with increase in \( \alpha \).

The variation of pressure rise with time averaged flow rate for different values of Hartmann number \( M \) is shown in Fig.3. It is observed that for a given \( \bar{Q}, \Delta p \) decreases for a decreasing \( M \) in pumping and free pumping regions. For a given \( \Delta p \) the flux \( \bar{Q} \) depends on \( M \) and it increases with increasing \( M \). The variation of pressure rise with time averaged flow rate for different values of micropolar parameter \( M \) is shown in Fig.4. It is observed that for a given \( \bar{Q}, \Delta p \) increases for a increasing \( m \) in pumping and free pumping regions. For a given \( \Delta p \) the flux \( \bar{Q} \) depends on \( m \) and it increases with increasing \( m \).

The effect of the inclination angle \( \theta \) on pumping characteristics is shown in Fig.5. It is observed that for a given \( \bar{Q}, \Delta p \) increases as the angle of inclination \( \theta \) increases. Also for a given \( \theta, \Delta p \) increases as \( \bar{Q} \) increases. The variation of pressure rise with time averaged flow rate for different values of Darcy number \( Da \) is shown in Fig.6. It is observed that for a given \( \bar{Q}, \Delta p \) increases for a increasing in \( Da \) pumping and free pumping regions. For a given \( \Delta p \) the flux \( \bar{Q} \) depends on \( Da \) and it increases with increasing \( Da \).

The effect of coupling parameter on the pumping characteristics is shown in Fig.7. We observed that the large the coupling number \( N \), the pressure rises against which the pumping works. For a given \( \bar{Q} \) the pressure difference increases with increase in \( N \). The effect of amplitude ratio \( \eta \) on pumping characteristics is shown in Fig.8. It is observed that the large the amplitude ratio, the greater the pressure rise against which the pump works. For a given \( \Delta p \), the flux \( \bar{Q} \) depends on \( h \) and it increases with increasing \( h \).
The effect of \( \eta \) on pumping characteristics is shown in Fig. 9. It is observed that for a given \( Q, \Delta p \) increases as \( \eta \) increases. Also for a given \( \eta \), \( \Delta p \) increases as \( Q \) increases. Figs. 10 to 14 are drawn to study the effect of various parameters on the microrotation velocity. From Fig. 10 it is observed that an increase in the slip parameter decreases the microrotation velocity. From Fig. 11 it is noticed that decrease in the darcy number decreases the microrotation velocity. From Fig. 12 it is observed that increase in \( M \) increases the microrotation velocity. Similarly increase in coupling parameter increases the microrotation velocity and is shown in figure. The effect of micropolar parameter on the microrotation velocity is shown in Fig. 13. It can be seen that the decrease in \( m \) decreases the microrotation velocity.

VI. CONCLUSIONS

Mathematical modeling of the peristaltic pumping of a Micropolar fluid under the effect of a magnetic field in an inclined channel is done in this paper. The following are the conclusions drawn from this.

1. Pumping decreases as the slip parameter \( \alpha \) increases in the pumping and free pumping regions. The opposite behavior is observed in co-pumping region.
2. For a given time averaged flow rate, the pressure difference decreases for a decreasing magnetic number.
3. The increase in micropolar parameter, increases the pumping in all the pumping regions. The same phenomenon is observed for the angle of inclination, Darcy number also.
4. The effect of various parameters on the microrotation velocity is studied. An increase in the slip parameter decreases the microrotation velocity. A decrease in the Darcy number decreases the microrotation velocity. An increase in the magnetic parameter increases the microrotation velocity. Similarly increase in coupling parameter increases the microrotation velocity.
Fig. 4. Variation of $\Delta p$ with $\bar{Q}$ for different values of micropolar parameter $m$.

Fig. 5. Variation of $\Delta p$ with $\bar{Q}$ for different values of angle of inclination $\theta$.

Fig. 6. Variation of $\Delta p$ with $\bar{Q}$ for different values of Darcy number $Da$. 
Fig. 7. Variation of $\Delta p$ with $Q$ for different values of coupling parameter $N$.

Fig. 8. Variation of $\Delta p$ with $Q$ for different values of $h$.

Fig. 9. Variation of $\Delta p$ with $Q$ for different values of $\eta!$.
Fig. 10. Variation of $\Omega$ with $y$ for different values of slip parameter $\alpha$.

Fig. 11. Variation of $\Omega$ with $y$ for different values of Darcy number $Da$.

Fig. 12. Variation of $\Omega$ with $y$ for different values of slip parameter $\alpha$. 
Fig. 13. Variation of $\Omega$ with $y$ for different values of coupling parameter $N$.

Fig. 14. Variation of $\Omega$ with $y$ for different values of micropolar parameter $m$.

REFERENCES


**BIOGRAPHIES:**

* S.V.H.N.Krishna Kumari,P. is a gold medalist in M.Sc(Applied Mathematics) from S.P.M.V.V., Tirupati. She obtained her Ph.D from Osmania University, Hyderabad. Her area of research is Mathematical modeling of Physiological flows. She has 15 international research publications to her credit. Presently she is working as Professor (Mathematics),Aurors Scientific Technological and Research Academy, Bandlaguda, Hyderabad.

* Saroj D Verneker completed M.Sc, M.Phil from Bangalore University, Bangalore.She working as Asst.Professor(Mathematics),Methodist College of Engineering and Technology, Abids,Hyderabad. She is pursuing her Ph.D from JNTU, Hyderabad.

* Y.V.K.Ravi Kumar did his M.Sc (Applied Mathematics) from S.V.University,Tirupati and Ph.D from Osmania University, Hyderabad. He is having 20 years of experience in teaching Mathematics and Computer Science courses. His research interests are Biofluid flows, Flow through porous media and speech recognition. He published 23 research papers in international journals. He is an editorial board member of international journals JETEAS, IJMES.