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Numerical Solution of Nonlinear Biochemical Reaction Model Using Variational Iteration Method

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ABSTRACT
In this paper, the variational iteration method was used to obtained numerical solution of Michaelis-Menten nonlinear biochemical reaction system. The correctional function for the system of equations were obtained and consequently the solution was obtained iteratively. Results obtained were compared with that of differential transform method (DTM) and were found to be in agreement.

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I. INTRODUCTION

The variational iteration method as proposed in [1,2] which is a modified general Lagrange multiplier method[3] has been shown to solve effectively, easily and accurately, a large class of nonlinear problems with approximation which converges quickly. It was successfully applied to autonomous ordinary differential equation [4], nonlinear partial differential equation with variable coefficient[5], Schrodinger-KDV, generalized KDV and shallow water equations[6], Burger's and coupled Burger's equation[7], the linear Helmotz partial differential equation [8] and recently to nonlinear fractional differential equation with Caputo differential derivative[9].

1.1 Mathematical model

Consider the model

$$E + A \Leftrightarrow Y \rightarrow E + X \tag{1}$$

As provided in [10]

Where E is the enzyme, A the substrate, Y intermediate complex and X the product. The time evolution of equation (1) can be determined from the solution of the system of coupled nonlinear ordinary differential equations

$$\frac{dA}{dt} = -K_1 EA + K_{-1} Y , \tag{2}$$

$$\frac{dE}{dt} = -K_1 EA + (K_{-1} + K_2)Y , \tag{3}$$

$$\frac{dY}{dt} = K_1 EA - (K_{-1} + K_2)Y , \tag{4}$$

$$\frac{dX}{dt} = K_2 Y , \tag{5}$$

Subject to the initial conditions

$A(0) = A_0, E(0) = E_0, Y(0) = Y_0, X(0) = X_0,$ where the parameters K_1, K_{-1}, K_2 are positive rate constants for each reaction. System (2)-(5) can be reduced to only two equations for A and Y and in dimensionless form of concentrations of substrate, x, and intermediate complex between enzyme and substrate, y, [10]

$$\frac{dx}{dt} = -x + (\beta - \alpha) y + xy \tag{6}$$

$$\frac{dy}{dt} = \frac{1}{\varepsilon}(x - \beta y - xy) \tag{7}$$

Subject to initial conditions

$$x(0) = 1, y(0) = 0 \quad \text{for the case } \alpha = 0.375, \beta = 1.0 \text{ and } \varepsilon = 0.1 \tag{8}$$

Where α, β and ε are dimensionless parameters.

II. VARIATIONAL ITERATION METHOD

Inokuti[3] proposed a general Lagrange multiplier method for solving nonlinear differential equations where he used nonlinear operator of the form as follows:

$$LU + NU = g(x) \tag{9}$$

Where L is a linear operator, N is a nonlinear operator, $g(x)$ is a known analytic function and U is an unknown that to be determined.

The Inokuti method was modified by [4] which can be written as

$$U_{n+1}(x_0) = U_n(x_0) + \int_0^{x_0} \lambda\left(L\tilde{U}_n + N\tilde{U}_n - g\right)ds \tag{10}$$

Where U_0 is an initial approximation and $\tilde{U}_n$ is a restricted variation. For arbitrary x_0

$$U_{n+1}(x) = U_n(x) + \int_0^{x} \lambda(LU_n(s) + NU_n(s) - g(s))ds \tag{11}$$

The above integral in equation (11) is called a correctional function and index n denotes the nth approximation. Also equation (11) is called variational teration method (VIM).With Lagrange multiplier λ [11]

$$\lambda = \frac{(-1)^m (s-x)^{m-1}}{(m-1)!} \tag{12}$$

Where m is the highest order of the given equation.

3 Application

Consider equations (6) and (7). Using equation (11), the correct functional for equation (6) and (7) are

$$X_{n+1} = X_n + \int_0^t \lambda[X_n' + X_n - (\beta - \alpha)Y_n - X_nY_n]dx$$

$$Y_{n+1} = Y_n + \int_0^t \lambda[Y_n' - \frac{1}{\varepsilon}(X_n - \beta Y_n - X_nY_n)]dy \tag{13}$$

The Lagrange multiplier from (12) is

$$\lambda = -1 \tag{14}$$

Solving equation (13) and equation (14) we have

$$X_1 = 1 - t$$

$$Y_1 = 10t \tag{15}$$

$$X_2 = 1 - t + \frac{17.25t^2}{2} - \frac{10t^3}{3}$$

$$Y_2 = 10t - 105t^2 + \frac{100t^3}{3} \tag{16}$$

$$X(t) = 1 - t + 8.62500000t^2 - 63.08333333t^3 + 62.18750000t^4 - 194.45833333t^5 + 106.25000000t^6 - 15.87301587t^7$$

$$Y(t) = 10t - 105t^2 + 762.08333333t^3 - 819.79166670t^4 + 2077.91666700t^5 - 2020.83333333t^6 + 476.19047620t^7 \tag{17}$$

The numerical solution above are in agreement with that of the differential transformation method (DTM) stated by [10] which clearly indicate variational iteration method as a powerful tool.

III. CONCLUSION

Variational iteration method was presented for Michaelis-Menten nonlinear biochemical reaction system. The method was found to be easy and efficient. Result obtained were compared with differential transformation method and are in agreement. The method is found to be accurate.

REFERENCES

[1.] J.H. He,A new approach to nonlinear partial differential equations, Comm. Nonlinear Sci. Numer. Simul. 2(4)(1997)230-235.

[2.] J.H. He, A variation approach to nonlinear problems and its application. Mech. Applic. 20(1)(1998)30-34.

[3.] M. inokuti et al. general use of the Lagrange multiplier in nonlinear Mathematical Physics, S.Nemat-Nassed, ed., Variational method in the Mechanics of solid Pergamon Press,(1978)156-162.

[4.] J.H. He, Variational iteration method for autonomous ordinary differential systems. Appl. Math.Comput. 118(2000)115-123.

[5.] J.H. He, Variational principle for some nonlinear partial differential equation with variable coefficients, Chaos, Solitons and Fractals 19(2004)847-851.

[6.] M.A Abdou and A.A. Soliman, New application of variational iteration method, Physica D211 (2005)1-8.

[7.] M.A Abdou and A.A. Soliman, Variational iteration method for solving Burger's and coupled Burgar's equations, J. Comput Appl. Math. 181(2005)245-251.

[8.] S. Momani and S. Abuasad, Application of He's variational iteration method to Helmotz equation, Chaos, Solitons and Fractals 27(2006)1119-1123.

[9.] Z.M. Odibat and S. Momani, Application of variational iteration method to nonlinear differential equation of fractional order, Int. J. Nonlinear Sci.Numer. Simul. 7(2006)27-36.

[10.] A. Batiha and B. Batiha, Differential transformation method for a reliable treatment of the nonlinear biochemical reaction model, Advanced studies in Biology, Vol.3 (2011)355-360.

[11.] S. Abbasbandy and E. Shivanian, Application of the variation iteration method for system of nonlinear Volterra integro-differential equations. Mathematical and computational applications, Vol. 14(2009)147-158.