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Studies of Effects of Convective Plane Stagnation Point Mhd Flow with Convective Boundary Conditions In The Presence Of A Uniform Magnetic Field

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Abstract

A numerical investigation is made on the convective plane stagnation point flow with convective boundary conditions in the presence of a uniform magnetic field. The system of coupled partial differential equations obtained from the modelled fluid flow was first transformed into a system of coupled ordinary differential equations with the use of similarity transformation. The ordinary differential equations are solved using fourth order Runge-Kutta formula together with shooting technique. The features of the flow and heat transfer characteristics for different values of the relevant flow quantities in engineering are investigated. Under this study, the Nusselt number and skin friction coefficient are obtained numerically and the results compared with those in the literature. It is seen that the magnetic field has a significant influence on the skin friction as the skin friction decreases with increasing values of the magnetic parameter whereas the combined influence of pressure gradient and magnetic fields invokes minimal changes on the Nusselt number .

Keywords: *plane stagnation point, convective boundary conditions, magnetic field*

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I. Introduction

The flow near a stagnation point has over the years become an interesting area of research both to scientists and investigators due to its overly wide scientific and industrial applications. It has found its usefulness in the cooling of electronics devices, nuclear reactors , extrusion of polymers, etc. Crane[1] studied the two dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate. Ramachandran[2] studied laminar mixed convection in two-dimensional stagnation flows around heated surfaces by considering both cases of an arbitrary wall temperature and arbitrary heat flux variations. Their results showed that for a specified range of buoyancy parameter, dual solutions existed and a reversed flow developed in the buoyancy opposing flow region. For an unsteady state and a vertical surface immersed in a micropolar fluid, studied by Devi[3] and Lok[4] respectively, dual solutions were found to exist for a specific range of buoyancy parameter. On the flow with convective boundary conditions, Aziz[5] studied a similarity solution applied to laminar thermal boundary layer flow over flat plate with a convective surface boundary conditions but only obtained local Biot numbers which was made global on restricted conditions. Okedayo[6] presented the effects of viscous dissipation on the mixed convection heat transfer over a flat plate with internal heat generation and convective boundary conditions, their studies also obtained local similarity variables. Okedayo[7] presented the similarity solution to the plane stagnation point flow with convective boundary conditions but the effect of the magnetic field on the flow has been neglected. Our research crux is based on including this relevant factor.With the above motivation, we present in this paper similarity solution to the plane stagnation point flow with convective boundary conditions and obtained global Biot numbers. This study may be regarded as the extension of [7]. It is to be pointed out, here that not many researchers in the resent past have included the convective boundary conditions in their investigations.

II. Mathematical Formulation

We consider two- dimensional, steady stagnation point flow of an electrically conducting, incompressible viscous MHD stream in which a vertical plate is immersed. The velocity of the laminar Boussinesq fluid external to the boundary layer is directed normal to the plate, and is moved along in the boundary layer with velocity field ($u(x, y)$, $v(x, y)$, x and y are coordinates along and normal to the plane. It is assumed that the plate is impermeable and the magnetic Reynolds number is negligibly small so that the induced magnetic fields effects can be ignored. The flow and heat transfer equations relevant for the model , namely the continuity, momentum and energy equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \gamma\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}B_0^{\,2}u \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

Boundary Conditions:

$$u(x,0) = v(x,0) = 0 \qquad (4a)$$

$$u(x,\infty) = ax = u_\infty \qquad (4b)$$

$$-k\frac{\partial T}{\partial y}(x,0) = h_f\left(T_f - T(x,0)\right) \qquad (4c)$$

$$T(x,\infty) = 0 \qquad (4d)$$

In equations (**1**)-(**4d**) the velocity field is $(u(x,y), v(x,y))$ where u, v are the components of the velocity along and normal to the plate respectively. T(x,y) is the temperature field, γ is the kinematic viscosity, α is the thermal diffusivity of the fluid, h_f is the heat transfer coefficient, p is the fluid pressure, ρ is the fluid density, k is the thermal conductivity, σ is electrical conductivity of the fluid, **B**= $(0, B_0)$ is the imposed magnetic field , a is the stretching rate constant and θ is dimensionless fluid temperature.

In order to transform the above equations, we introduce the following similarities variables as follows:

Similarity Variables / Dimensionless Numbers

$$\left.\begin{array}{ll} \eta = y\sqrt{\frac{a}{\gamma}}, & \psi(x,y) = x\sqrt{a\gamma}\,f(\eta) \\ \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, & Pr = \frac{\gamma}{\alpha} \\ M = \frac{\sigma B_0^{\,2}}{\rho a}, & Bi = \frac{h_f}{k}\sqrt{\frac{\gamma}{a}} \end{array}\right\} \qquad (5)$$

Here, Pr, M, Bi signifies respectively the Prandtl number, the magnetic parameter, and the Biot number.

In terms of the stream function $\psi(x,y)$, the velocity components are

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{d\psi}{dx} \qquad (6)$$

which in essence satisfy (**1**) identically.

With the use of the similarity transformation, momentum and energy equations become

$$f''' + ff'' - (f')^2 + 1 - Mf' = 0 \qquad (7)$$

$$\theta'' + P_r f\theta' = 0 \qquad (8)$$

Subject to the transformed boundary conditions

$$f'(0) = f(0) = 0, \; f'(\infty) = 1 \qquad (9)$$

$$-\theta'(0) = Bi\left(1 - \theta(0)\right), \theta(\infty) = 0 \qquad (10)$$

Where the prime denotes differentiation with respect to η, $P_r = \frac{\gamma}{\alpha}$ is the Prandtl number.

The system of equations is a coupled system of non-linear differential equations and it is difficult to solve by the common methods of the system of ordinary differential equations.

III. Numerical Results And Discussion

The non-linear equations are solved using a classical fourth order Runge-Kutta method with a shooting technique implemented on a computer programme written in Maple(15). A convenient step size was chosen to obtain the desired accuracy. Figure 1 shows the numerical results for fixed Biot and Prandtl number of 0.05 and0.71 respectively fordifferent values of Magnetic parameter M=0.1, 0.4, 0.2 and 0.3. It is observed that as the magnetic parameter increases, the velocity boundary layer thickness decreases. Furthermore, the velocity decreases.Figure 2 shows the effects of Biot number Bi = 0.05, 0.1, 5 and 20 for constant Prandtl number Pr = 0.71 and Magnetic parameter M= 0.1. We observe that an increase in the Biot number increases the thermal boundary layer thickness and also increases the wall temperature.Figure 3 shows the effects of Prandtl number

Pr = 0.71, 1, 3 and 7.1 for constant values of Biot number Bi = 0.1and Magnetic parameter M = 0.1. We observe that as the Prandtl number increases, the thermal boundary layer thickness decreases. Similarly, the surface temperature decreases.

Figures 2 and 3 also show that the boundary conditions are satisfied. In table 1, we show the values of the Skin Friction Coefficient as being influenced by various values of Magnetic parameter, Biot number and Prandtl number. The temperature at the wall is also shown in the last column. It is observed that an increase in the Magnetic parameter cause a reduction in the Skin Friction Coefficient. In table 2, we present the computed values of Nusselt number. A numerical comparison is also made with the results of Okedayo et al (2012) and that of a flat plate and it is observed that the introduction of the magnetic field to the stagnation point flow reduces Nusselt number slightly but it is still higher than that of a flat plate.

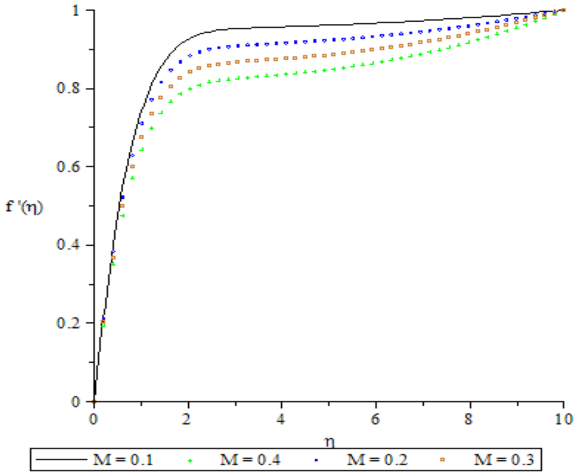

Figure 1: Effects of magnetic parameter on the velocity profiles for Bi = 0.05 and Pr = 0.71

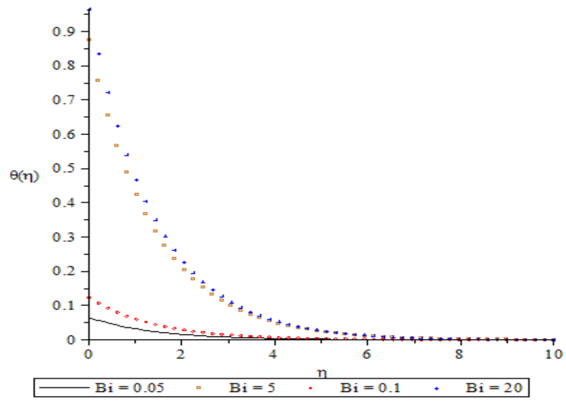

Figure 2: Effects of Biot number on the temperature profiles for M = 0.1 and Pr = 0.71

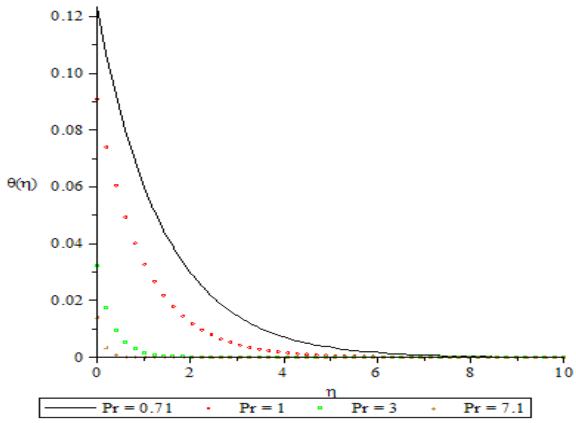

Figure 3: Effects of Prandtl number (Pr) on the temperature profiles when M = 0.1 and Bi = 0.1

Table 1: Computations showing the influence of M, Bi andPr on the Skin friction coefficient

M	Bi	Pr	$f''(0)$
0.1	0.1	0.71	1.18258392054224
0.2	0.1	0.71	1.13586145675196
0.4	0.1	0.71	1.05160957361730
0.1	5	0.71	1.18258392054289
0.1	20	0.71	1.18258392054274
0.1	∞	0.71	1.18258392054268
0.1	0.1	3	1.18258392053607
0.1	0.1	7.1	1.18258392053607

Table 2: Comparison of various Nusselt numbers

M	Bi	Pr	Okedayo [7]	Flat plate	This study
0.1	0.05	0.71	0.0454	0.0429	0.04532
0.1	0.2	0.71	0.1428	0.1204	0.14195
0.1	0.4	0.71	0.2220	0.1723	0.22004
0.1	0.8	0.71	0.3072	0.2195	0.30352
0.4	0.05	0.71	0.0454	0.0429	0.04511
0.4	0.05	3.0	0.0473	0.0453	0.04708
0.4	0.05	7.0	0.0480	0.0464	0.04783
0.4	0.05	11.4	0.0483	0.0469	0.048169

IV. Conclusion

The problem of plane stagnation point has been solved using the method of similarity transformation together with the Runge-Kutta method of fourth orther and shooting technique. It is observed that the convective conditions, the stagnation point and the Magnetic field have significant effects on the heat transfer rate, velocity boundary layer thickness and the thermal boundary layer thickness.

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