Efficient and robust denoising of magnetic resonance brain images based on wavelet based nonlocal means algorithm

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ABSTRACT

Image denoising has become an essential exercise in medical imaging especially the Magnetic Resonance Imaging (MRI). In the proposed method noisy image is first decomposed into subband by wavelet transform and the nonlocal means filter is applied to each subband. This proposed method preserves the wavelet coefficients corresponding to the structures, while effectively suppressing noisy ones. Experimental results are also compared with the other different techniques like median, wiener, wavelet, wavelet based wiener, non-local mean. The quality of the output images is measured by the statistical quantity measures such as peak signal-to-noise ratio (PSNR), signal-to-noise ratio (SNR) and Mean square error (MSE). The quantitative and the qualitative measures used as the quality metrics demonstrate the ability of the proposed method for noise suppression.

KEYWORDS: Denoising, Wavelet, MRI, Wiener filtering, NLM.

I. INTRODUCTION

Magnetic Resonance Images (MRI) are widely used for diagnosis and the treatment of brain tumors. It is the most powerful imaging technique developed to study the structural features and the functional characteristics of the internal body parts. It possesses good contrast resolution for different tissues and has advantages over computerized tomography (CT) for brain tissue studies. The diagnostic and visual quality of the MR images are affected by the noise added while acquisition. Noise in MRI [1] is mainly due to thermal noise that is induced by the movement of the charged particles in the radio frequency coils as well as the small anomalies in the preamplifiers. The presence of noise not only produces undesirable visual quality but also lowers the visibility of low contrast objects. Noise removal is essential in medical imaging applications in order to enhance and recover anatomical details that may be hidden in the data. In recent years, wavelet transform shows a clear advantage in the field of signal and image denoising domains, and has many research results. The important property of a good image-denoising model is that it should completely remove noise as far as possible as well as preserve edges. In this paper effectiveness of six denoising algorithms viz. median filter [2], wiener filter [3], wavelet filter [4], wavelet based wiener [5], NLM [6], wavelet based NLM [7] using MRI images in the presence of additive white Gaussian noise is compared. Wavelet filter [4] removes noise pretty well in smooth regions but perform poorly along the edges. Image denoising has been extensively studied and thus there is a large amount of literature on denoising. Among these numerous works, we will briefly mention only a few of recently developed methods that are related with our method, specifically the wavelet domain coefficient thresholding and modeling [8],[9],[10] and nonlocal means filter [6]. The problem with the conventional wavelet domain filtering is the removal of small but important coefficients while thresholding or the generation of unwanted coefficients in the probabilistic modeling approach as stated above. In this paper, it is expected that the nonlocal means filtering of the coefficients can alleviate these problems while effectively removing noisy coefficients. Specifically we propose a wavelet domain image denoising method where the nonlocal means filtering is applied to each of the subbands. By the nonlocal means filtering, the small wavelet coefficients which are part of important image structures are well kept while suppressing the noisy coefficients, whereas the conventional wavelet denoising methods sometimes suppress small but important coefficients as well.
II.  DENOISING OF MR IMAGES

Image denoising is a common pre-processing step in many Magnetic Resonance (MR) image processing and analysis tasks, the goal of denoising is to remove the noise, which may corrupt an image during its acquisition or transmission, while retaining its quality. Noise in MR images obeys a Rician distribution [1]. In contrast to Gaussian additive noise, Rician noise is signal dependent and is therefore more difficult to separate from the signal. For low SNR, the Rician distribution tends to the Rayleigh distribution. For high SNR the Rician distribution tends to the Gaussian distribution. In practice, the Rician distribution can be well approximated by a Gaussian one for SNR values. The performance measures for denoising schemes are expressed as follows.

\[ MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [l(i,j) - K(i,j)]^2 \]  
\[ PSNR_{dB} = 10 \log_{10} \left( \frac{MAX^2}{MSE} \right) \]  

III.  DENOISING ALGORITHMS

In this section, we explain the basic principles of denoising algorithms such as median, wiener, Wavelet, wavelet based wiener and Non-Local Means used in our paper.

A.  Median filter

The median filter is a popular nonlinear digital filtering technique, often used to remove noise. Such noise reduction is a typical pre-processing step to improve the results of later processing (for example, edge detection on an image). Median filtering is very widely used in digital image processing because under certain conditions, it preserves edges while removing noise [2]. Sometimes known as a rank filter, this spatial filter suppresses isolated noise by replacing each pixel’s intensity by the median of the intensities of the pixels in its neighbourhood. It is widely used in de-noising and image smoothing applications. Median filters exhibit edge-preserving characteristics (Linear methods such as average filtering tends to blur edges), which is very desirable for many image processing applications as edges contain important information for segmenting, labelling and preserving detail in images. This filter may be represented by Equation.

\[ G(u,v) = \text{median}\{I(x,y),(x,y) \text{wF}\} \]  
Where
\[ \text{wF} = w \times w \] Filter window with pixel (u, v) as its middle.

It often fails to perform well as linear filters in providing sufficient smoothing of non impulsive noise components such as additive Gaussian noise. One of the main disadvantages of the basic median filter is that it is location-invariant in nature, and thus also tends to alter the pixels not disturbed by noise.

B.  Wiener filter

Wiener filters are a class of optimum linear filters which involve linear estimation of a desired signal sequence from another related sequence. It is not an adaptive filter. The Wiener filter’s main purpose is to reduce the amount of noise present in a image by comparison with an estimation of the desired noiseless image. The Wiener filter may also be used for smoothing. This filter is the mean squares error-optimal stationary linear filter for images degraded by additive noise and blurring. It is usually applied in the frequency domain (by taking the Fourier transform) [3], due to linear motion or unfocussed optics Wiener filter is the most important technique for removal of blur in images. From a signal processing standpoint. Each pixel in a digital representation of the photograph should represent the intensity of a single stationary point in front of the camera. Unfortunately, if the shutter speed is too slow and the camera is in motion, a given pixel will be an amalgram of intensities from points along the line of the camera's motion. The goal of the Wiener filter is to filter out noise that has corrupted a signal. It is based on a statistical approach. Typical filters are designed for a desired frequency response. The Wiener filter approaches filtering from a different angle. One is assumed to have knowledge of the spectral properties of the original signal and the noise, and one seeks the LTI filter whose output would come as close to the original signal as possible. Wiener filters are characterized by the following:

1. Assumption: signal and (additive) noise are stationary linear random processes with known spectral characteristics.
2. Requirement: the filter must be physically realizable, i.e. causal (this requirement Can be dropped, resulting in a non-causal solution).
\[ G(u, v) = \frac{H^*(u, v)P_s(u, v)}{|H(u, v)|^2P_s(u, v)+P_n(u, v)} \]  

Where

- \( H(u, v) \) = Fourier transform of the point spread function
- \( P_s(u, v) \) = Power spectrum of the signal process, obtained by taking the Fourier transform of the signal autocorrelation
- \( P_n(u, v) \) = Power spectrum of the noise process, obtained by taking the Fourier transform of the noise autocorrelation

It should be noted that there are some known limitations to Wiener filters. They are able to suppress frequency components that have been degraded by noise but do not reconstruct them. Wiener filters are also unable to undo blurring caused by band limiting of \( H(u, v) \), which is a phenomenon in real-world imaging systems.

C. Wavelet Filter

Wavelet transform shows a clear advantage in the field of signal and image denoising domains, and has many research results [4]. The classical wavelet-based denoising method is proposed by Donoho et al. Steps for implementing denoising using wavelet based soft thresholding technique are as follows:

- **Calculate two-level Daubechies wavelet transform of the noisy image**
- **Modify the noisy wavelet coefficients according to soft thresholding rule**

  Where Donoho threshold also called Universal threshold given by:

  \[ t_u = \hat{\sigma} \sqrt{2 \log(n)} \]  

  where \( n \) is the number of wavelet coefficients, and \( \hat{\sigma} = \frac{\text{MAD}}{0.6745} \) is the estimates of the noise standard deviation. MAD denotes the Median Absolute Deviation of the wavelet coefficients in the finest resolution level. The wavelet coefficients \( w_{j,k} \) above the universal threshold are updated by soft thresholding: \( \text{sgn} (w_{j,k}) (|w_{j,k}| - t_u) \) in practical applications, the variance of the noise is estimated by dividing the MAD by 0.6745.

- **Compute the inverse Daubechies wavelet transform using modified coefficients and then get denoised image.**

D. Non-local Means Filter

The non-local means algorithm does not make the same assumptions about the image as other methods. Instead it assumes the image contains an extensive amount of self-similarity. The self-similarity assumption can be exploited to denoise an image. Pixels with similar neighborhoods can be used to determine the denoised value of a pixel.

Each pixel \( p \) of the non-local means denoised image is computed with the following formula:

\[ \text{NL}(V)(p) = \sum_{q \in N_i} w(p, q) V(q) \]  

Where \( V \) is the noisy image, and weights \( w(p, q) \) meet the following conditions \( 0 \leq w (p, q) \leq 1 \) and \( \sum w(p, q) = 1 \). Each pixel is a weighted average of all the pixels in the image. The weights are based on the similarity between the neighbourhoods of pixels \( p \) and \( q \). If \( w(p, q1) \) is much greater than \( w(p, q2) \) because pixels \( p \) and \( q1 \) have similar neighbourhoods and pixels \( p \) and \( q2 \) do not have similar neighbourhoods. In order to compute the similarity, a neighbourhood must be defined. Let \( N_i \) be the square neighbourhood centered about pixel \( i \) with a user-defined radius \( R_{sim} \). To compute the similarity between two neighborhoods take the weighted sum of squares difference between the two neighborhoods or as a formula \( d(p, q) = (V(N_p) - V(N_q)) F \). \( F \) is the neighborhood filter applied to the squared difference of the neighborhoods and will be further discussed later in this section. The weights can then be computed using the following formula:

\[ w(p, q) = \frac{1}{Z(p)} e^{-d(p, q) h} \]  

\( Z(p) \) is the normalizing constant defined as \( Z(p) = \sum e^{-d(p, q) / h} \). \( h \) is the weight-decay control parameter. As previously mentioned, \( F \) is the neighborhood filter with radius \( R_{sim} \). The weights of \( F \) are computed by the following formula:
Where $m$ is the distance the weight is from the center of the filter. The filter gives more weight to pixels near the center of the neighborhood, and less weight to pixels near the edge of the neighborhood. The center weight of $F$ has the same weight as the pixels with a distance of one [6]. Despite the filter's unique shape, the weights of filter $F$ do sum up to one.

### E. Wavelet Domain Wiener Filtering

In the filter we assume that the wavelet coefficients are conditionally independent Gaussian random variables. The noise is also modelled as stationary independent zero-mean Gaussian variable. Let us consider an image corrupted by a zero-mean Gaussian noise.

When using the Daubechies wavelet transform the steps for implementing denoising using the Wiener filter technique is as follows: 

1. Compute $\tilde{S}$ by convolving $\{y_{i,j}\}$ is the kernel of size 9.
2. The wiener filter is then applied using the formula
   \[
   \tilde{S} = \frac{\max(q_{i,j}-\sigma^2,0)}{q_{i,j}} y_{i,j} = \tilde{a}_{i,j} y_{i,j}
   \] (9)
3. Then apply inverse Daubechies wavelet transform.

### F. Proposed Wavelet Domain Nonlocal Means Filter

1. Perform wavelet transform to the noisy image; apply non local means filtering for the wavelet domain.
2. Exploit the excellent localization property of the wavelet transform as demonstrated in the conventional wavelet domain denoising, while keeping the main coefficients and its neighbors (structures) which might have been shrank in the conventional wavelet denoising.
3. By averaging structures similar as the current significant coefficient and its neighbors by the nonlocal means filtering, the structures are kept while the noisy coefficients are averaged out. Thus it is expected that the ringing artifacts would be alleviated compared to the conventional wavelet denoising while keeping the structures very well like the spatial domain nonlocal means filter.
4. Select adaptive bandwidth using the following equation
   \[
   h_0 = \left( \frac{4\delta^2}{3N} \right)^{1/5} \approx 1.06\delta N^{-1/5}
   \] (10)
   Where
   - $N$ - no of sample data
   - $\sigma$ - Standard deviation
5. Adaptively regulate the bandwidth in each sub band according to the relationship, $h = kho$, where $k$ is a scaling factor and $ho$ is the optimal band width
6. Finally then all the resultant coefficient is reconstructed by applying inverse wavelet transform which results in denoised image.

### IV. RESULTS AND DISCUSSION

#### A. Visual Quality Comparison

The experiments were conducted on two MRI datasets such as T1 weighted, T2 weighted MRI images, which are corrupted with additive white Gaussian noise in the clinical data sets, the images are acquired using Siemens Magnetom Avanto 1.5T Scanner.

- T2 weighted MR brain image with TR = 4000ms, TE = 114 ms, 5mm thick and 590×612 resolution.
- T1 weighted MR brain image with TR = 694ms, TE = 12 ms, 5mmThick and 630×645 resolution.
T1 weighted and T2 weighted brain MRI corrupted by additive white Gaussian noise and after denoising schemes are respectively shown in figures 1-8. These figures are provided the visual comparison of the results. From the results the wavelet based NLM filter perform better while compared with median, wiener, wavelet, wavelet based wiener and NLM filter.

![Filtered MRI brain images by adding additive white Gaussian noise with Noise (σ) =10](image)

B. Quantitative Comparison

The quantitative performances for filters are compared in terms of PSNR, SNR, and MSE. The quantitative performances in terms of PSNR, SNR and MSE for the various denoising algorithms are given in Table 1. From these results, as the noise level increases, significant change in the performance of the denoising results can be observed. Higher the value of PSNR and higher the value of SNR, lower the value of MSE shows that the wavelet based nlm filter perform superior than the other denoising methods.

**TABLE 1**

<table>
<thead>
<tr>
<th>Denoising schemes</th>
<th>Noise(σ)=10</th>
<th>Noise(σ)=20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSNR</td>
<td>SNR</td>
</tr>
<tr>
<td>Median Filter</td>
<td>11.47</td>
<td>7.5348</td>
</tr>
<tr>
<td>Wiener Filter</td>
<td>20.07</td>
<td>10.60</td>
</tr>
<tr>
<td>Wavelet filter</td>
<td>25.64</td>
<td>12.81</td>
</tr>
<tr>
<td>Wavelet based wiener</td>
<td>27.14</td>
<td>13.81</td>
</tr>
<tr>
<td>Non local means Filter</td>
<td>29.87</td>
<td>18.92</td>
</tr>
<tr>
<td>Proposed Wavelet Based NLM Filter</td>
<td>35.15</td>
<td>20.4929</td>
</tr>
</tbody>
</table>

![Chart-1: Analysis of PSNR and SNR values of various filter by adding additive white Gaussian noise of variance 10 and 20.](image)
V. SUMMARY AND CONCLUDING REMARKS

In this article, the performance comparison of various filtering methods for removing additive white Gaussian noise from MR images have been discussed. In this work T1 weighted, T2 weighted MRI brain images were used. The wavelet based nlm filtering method tends to produce good denoised image not only in terms of visual perception but also in terms of the quality metrics such as PSNR and SNR, MSE. Hence the new proposed algorithm based on the wavelet based nlm is found to be more efficient than the other methods in MR brain image denoising particularly for the removal of Gaussian noise. Thus the obtained results in qualitative and quantitative analysis show that this proposed algorithm outperforms the other methods both visually and in terms of PSNR, SNR, MSE.

REFERENCES: